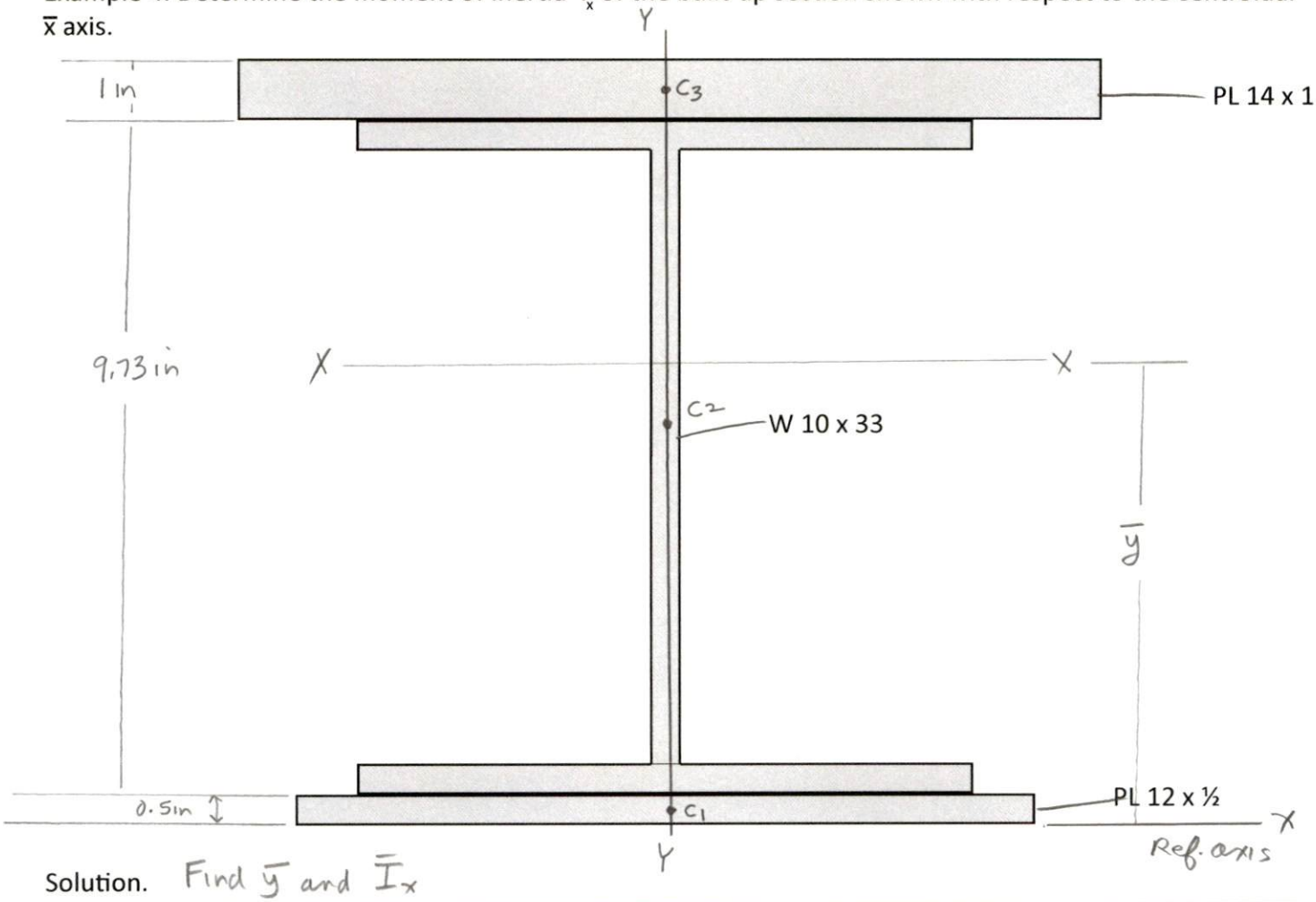
Example 4. Determine the moment of inertia $\bar{I}_x$ of the built-up section shown with respect to the centroidal $\bar{x}$ axis.



Solution. Find $\bar{y}$ and $\bar{I}_x$

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Part	A (in²)	y (in)	Ay (in³)	ȳ-y (in)	A(ȳ-y)² (in⁴)	I (in⁴)
PL 12x½	6	0.25	1.5	6.61	262.15	$\frac{12(0.5)^3}{12} = 0.125$
W10x33	9.71	5.365	52.0942	1.495	21.7	170
PL14x1	14	10.73	150.22	-3.87	209.68	$\frac{14(1)^3}{12} = 1.1667$
Σ	29.71		203.8142		493.53	171.2917

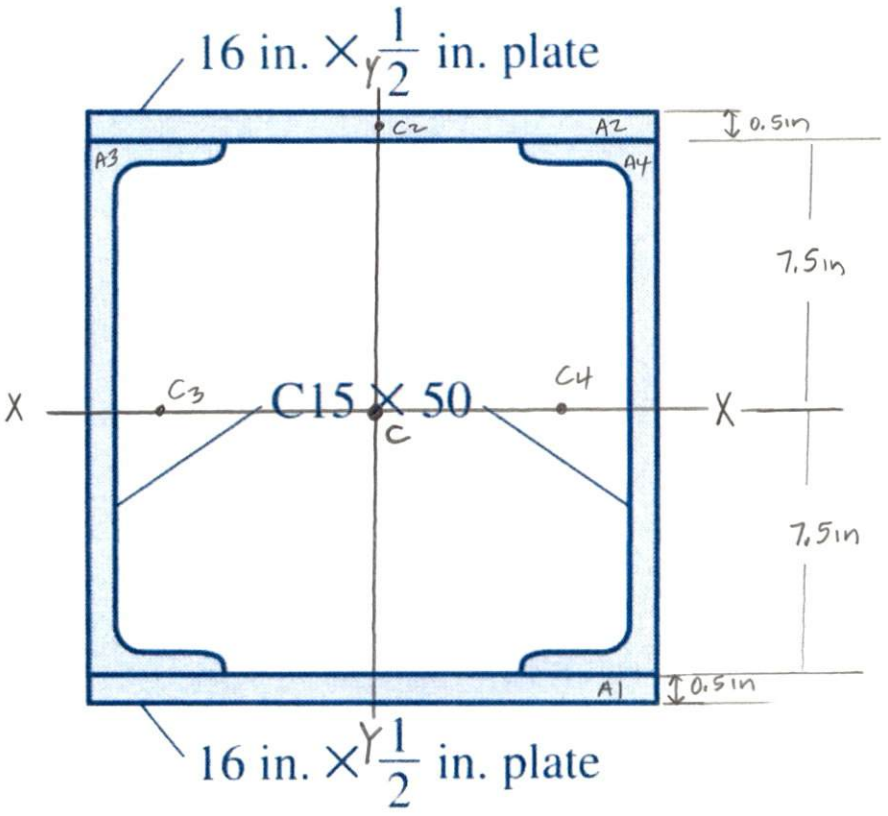
$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{203.8142\ in^3}{29.71\ in^2} = 6.86\ in$$

$$\bar{I}_x = \Sigma\left[I + A(\bar{y}-y)^2\right] = 171.2917\ in^4 + 493.53\ in^4$$

$$= 664.8\ in^4$$

Example 5

Problem 8-28. For the built-up section shown, determine the moment of inertia and the radius of gyration of the section with respect to the horizontal centroidal axis.



This Built-Up Section is symmetrical with respect to both centroidal axes.

Solution.

(1)	(2)	(3)	(4)	(5)
Part	A (in²)	y (in)	Ay² (in⁴)	I (in⁴)
PL 16 × 1/2	8	−7.75	480.5	$\frac{bh^3}{12}$ 0.167
PL 16 × 1/2	8	7.75	480.5	0.167
C 15×50	14.7	0	0	404
C15×50	14.7	0	0	404
Σ	45.4		961	808.33

$$\bar{I}_x = \Sigma [I + Ay^2] = 808.33\ in^4 + 961\ in^4 = \underline{1770\ in^4}$$

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{1770\ in^4}{45.4 in^2}} = \underline{6.24\ in}$$

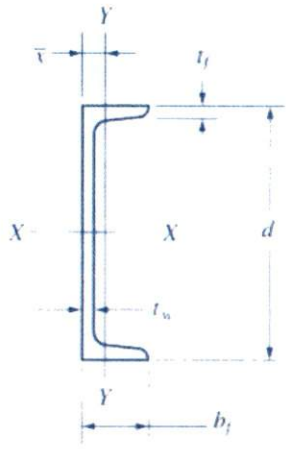


TABLE A–3(a) Properties of C Shapes (American Standard Channels): U.S. Customary Units

Designation	Area	Depth	Web	Flange		Axis x–x			Axis y–y			Centroid
			Thickness	Width	Average Thickness							
in. × lb/ft	A (in.^{2})	d (in.)	t_w (in.)	b_f (in.)	t_f (in.)	I (in.^{4})	S (in.^{3})	r (in.)	I (in.^{4})	S (in.^{3})	r (in.)	$\bar{x}$ (in.)
C15 × 50	14.7	15.00	0.716	3.716	0.650	404	53.8	5.24	11.0	3.78	0.867	0.798
× 40	11.8	15.00	0.520	3.520	0.650	349	46.5	5.44	9.23	3.37	0.886	0.777
× 33.9	9.96	15.00	0.400	3.400	0.650	315	42.0	5.62	8.13	3.11	0.904	0.787
C12 × 30	8.82	12.00	0.510	3.170	0.501	162	27.0	4.29	5.14	2.06	0.763	0.674
× 25	7.35	12.00	0.387	3.047	0.501	144	24.1	4.43	4.47	1.88	0.780	0.674
× 20.7	6.09	12.00	0.282	2.942	0.501	129	21.5	4.61	3.88	1.73	0.799	0.698
C10 × 30	8.82	10.00	0.673	3.033	0.436	103	20.7	3.42	3.94	1.65	0.669	0.649
× 25	7.35	10.00	0.526	2.886	0.436	91.2	18.2	3.52	3.36	1.48	0.676	0.617
× 20	5.88	10.00	0.379	2.739	0.436	78.9	15.8	3.66	2.81	1.32	0.692	0.606
× 15.3	4.49	10.00	0.240	2.600	0.436	67.4	13.5	3.87	2.28	1.16	0.713	0.634
C 9 × 20	5.88	9.00	0.448	2.648	0.413	60.9	13.5	3.22	2.42	1.17	0.642	0.583
× 15	4.41	9.00	0.285	2.485	0.413	51.0	11.3	3.40	1.93	1.01	0.661	0.586
× 13.4	3.94	9.00	0.233	2.433	0.413	47.9	10.6	3.48	1.76	0.962	0.669	0.601
C 8 × 18.75	5.51	8.00	0.487	2.527	0.390	44.0	11.0	2.82	1.98	1.01	0.599	0.565
× 13.75	4.04	8.00	0.303	2.343	0.390	36.1	9.03	2.99	1.53	0.854	0.615	0.553
× 11.5	3.38	8.00	0.220	2.260	0.390	32.6	8.14	3.11	1.32	0.781	0.625	0.571
C 7 × 14.75	4.33	7.00	0.419	2.299	0.366	27.2	7.78	2.51	1.38	0.779	0.564	0.532
× 12.25	3.60	7.00	0.314	2.194	0.366	24.2	6.93	2.60	1.17	0.703	0.571	0.525
× 9.8	2.87	7.00	0.210	2.090	0.366	21.3	6.08	2.72	0.968	0.625	0.581	0.540
C 6 × 13	3.83	6.00	0.437	2.157	0.343	17.4	5.80	2.13	1.05	0.642	0.525	0.514
× 10.5	3.09	6.00	0.314	2.034	0.343	15.2	5.06	2.22	0.866	0.564	0.529	0.499
× 8.2	2.40	6.00	0.200	1.920	0.343	13.1	4.38	2.34	0.693	0.492	0.537	0.511
C 5 × 9	2.64	5.00	0.325	1.885	0.320	8.96	3.56	1.83	0.632	0.450	0.489	0.478
× 6.7	1.97	5.00	0.190	1.750	0.320	7.49	3.00	1.95	0.479	0.378	0.493	0.484
C 4 × 7.25	2.13	4.00	0.321	1.721	0.296	4.59	2.29	1.47	0.433	0.343	0.450	0.459
× 5.4	1.59	4.00	0.184	1.584	0.296	3.85	1.93	1.56	0.319	0.283	0.449	0.457
C 3 × 6	1.76	3.00	0.356	1.596	0.273	2.07	1.38	1.08	0.305	0.268	0.416	0.455
× 5	1.47	3.00	0.258	1.498	0.273	1.85	1.24	1.12	0.247	0.233	0.410	0.438
× 4.1	1.21	3.00	0.170	1.410	0.273	1.66	1.10	1.17	0.197	0.202	0.404	0.436